# When Do the Seasons Start? 

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If you ask someone when winter starts, the response will likely be: December $22^{\text {nd }}$, the winter solstice. Similarly, summer would begin on the summer solstice (June $22^{\text {nd }}$ ) while spring and fall would start on the spring and fall equinox, March $22^{\text {nd }}$ and September $22^{\text {nd }}$, respectively. These dates are based on the orbit of the Earth around the Sun and have nothing to do with the annual temperature cycle. These dates define the astronomical seasons.

If you ask a meteorologist when summer and winter occur, the response would be June-July-August for summer and December-January-February for winter. The three warmest months represent summer while the three coldest months are winter. The transition periods of spring and fall are March-April-May and September-October-November, respectively. These periods define the meteorological seasons.

From a more intuitive perspective, you would expect summer to represent the warmest quarter of the year while winter would be the coolest quarter, with neither season confined by monthly calendar designations. This raises the question: How can we determine the dates of the seasons based quarterly periods?

Figure 1 show the annual temperature cycle for the maximum temperature at Kansas City International Airport (KMCI) based on daily normal high temperatures. The curve peaks out at $89^{\circ} \mathrm{F}$ from July $8^{\text {th }}$ through August $7^{\text {th }}\left(31\right.$ days) and bottoms out at $35^{\circ} \mathrm{F}$ from January $10^{\text {th }}$ through $17^{\text {th }}$ ( 8 days). The annual temperature cycle looks like a sine wave. This suggests a mathematical approach to answering the question posed above: harmonic analysis.

Harmonic analysis is a "statistical method for determining the amplitude and period of certain harmonic or wave components in a set of data with the aid of Fourier series" (Glickman, 2000). This technique allows a data series to be decomposed into a series of sine and cosine waves that, when added together, approximate the original series. The wave components are defined in terms of the series length. The component with the longest wavelength corresponds to the length of the series, i.e., the
longest wavelength equals the length of the series. The second longest wavelength equals one-half of the series length; the third longest wavelength equals one-third of the series length; etc. The wavelength with the largest amplitude implies a strong periodicity at that wavelength; low amplitudes imply weak periodicities at those wavelengths.


Figure 1: Annual maximum temperature at Kansas City International Airport based on daily normals.

Data used to construct Figure 1 are the daily normal high temperatures for Kansas City International Airport. These data produce a time series of 366 days from January $1^{\text {st }}$ through December $31^{\text {st }}$ (including February $29^{\text {th }}$ ). Using the equations described by Jenkins and Watts (1968), these data were used to decompose the curve in Figure 1 into a series of cosine harmonics. The amplitude and phase of these harmonics are listed in the table below.

The m column in Table 1 is the harmonic number. The average high temperature for the year is shown as the amplitude for $m=0$ : $64.312^{\circ}$ F. For the remaining harmonics, $m=1$ has the largest amplitude. This indicates that the annual temperature cycle is the dominant component in Figure 1.

Using the center of the warmest and coolest periods noted above, the highest average maximum temperature at KMCI occurs around July $23^{\text {rd }}$ while the coldest maximum temperature is on January $13^{\text {th }}-14^{\text {th }}$. These dates are about 191 days apart, longer than the 183 day half-wavelength of the first harmonic. As a result, the second harmonic ( $m=2$ ) has a small, but significant, amplitude.

| $\mathbf{m}$ | Amplitude | Phase |
| :---: | :---: | :---: |
| 0 | 64.312 |  |
| 1 | 12.893 | -15.653 |
| 2 | 1.390 | 14.578 |
| 3 | 0.530 | -24.106 |
| 4 | 0.101 | 4.290 |
| 5 | 0.095 | 19.619 |

Table 1: Amplitude and phase of harmonics
for KMIC maximum temperatures
The value in the phase column indicates where the peak of the cosine curve occurs. If you translate the -15.653 for $m=1$ into a day of the year it yields Julian day 199.9 or July $18^{\text {th }}$ (day 200) for the warmest day. Similarly, the coldest day would be 183 days earlier on Julian day 16.9 or January $17^{\text {th }}$ (day 17). Both of these dates occur within the periods of warmest and coolest temperatures.

The next order of business is to divide the first harmonic into four equal quarters based on the phase. This analysis yields the following dates for the seasons:

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Spring begins March 3 rd
Summer begins June 2 nd
Autumn begins September 2 nd
Winter begins December 2 nd
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From a temperature perspective, these dates are the beginning of the four seasons for Kansas City. Using these dates summer becomes the warmest quarter of the year while winter is the coldest quarter of the annual cycle.

Trenberth (1983) conducted a more elaborate statistical analysis based on monthly temperature data for much of the Earth using techniques similar to those outlined above. For the United States he found the following dates:

| Spring begins | March $4^{\text {th }}$ |
| :--- | :--- |
| Summer begins | June $4^{\text {th }}$ |
| Autumn begins | September $3^{\text {rd }}$ |
| Winter begins | December $3^{\text {rd }}$ |

His dates are for the 48 contiguous states and are very close to those found for the Kansas City data analyzed in this enote. He also pointed out that the concept of seasons is strictly true only in middle latitudes which have distinct warm and cold period and start dates vary somewhat from place to place.

So, the next time someone asks you when does summer start, say: "In Kansas City, it's June $2^{\text {nd }!" ~}$

## References

Glickman, Todd S., 2000: Glossary of Meteorology. American Meteorological Society, 855 pp.

Jenkins, Gwilym M., and Donald G. Watts, 1968: Spectral Analysis and its Applications. Holden-Day, 525 pp .

Trenberth, Kevin E., 1983: What are the seasons? Bulletin of American Meteorological Society, 64, 11, 1276-1282.

