Chapter 5

Derived Parameters

This chapter focuses on the definition and application of what we call derived parameters. A *derived parameter* is a dynamic, kinematic, or thermodynamic quantity that can be calculated from observed weather data or from forecast model output of basic weather data. These parameters have been found useful in evaluating the current and future state of the atmosphere. We will discuss only six quantities in this chapter even though many more exist. Thermodynamic-based derived parameters, based on sounding stability analysis, are discussed in Chapter 12.

Coordinate Systems: In discussing the derived parameters listed below, equations are used to mathematically define these parameters. In most cases a Cartesian coordinate system is used with x as the eastward axis and y as the northward axis of the horizontal plane, and either z (height) or p (pressure) as the vertical coordinate. The variables u, v, and w are used for the wind components in the x, y, and z directions, respectively. In some cases a natural coordinate system may be used where s is the direction along the flow and n is the direction perpendicular to the flow.

Vorticity

Vorticity is one of several kinematic properties of fluid flow. It is a three-dimensional vector that represents the local rotation of the flow. This vector can be decomposed into a vertical component and a horizontal component. Using the right-hand rule, the vertical component represents rotation in the horizontal plane, while the horizontal component is the rotation in the vertical plane. Mathematically, vorticity is the curl of the velocity vector.

Operational forecasters routinely use the vertical component (ζ) to assess horizontal flow rotation. The mathematical expression for this component is:

$$\zeta = (\partial v/\partial x) - (\partial u/\partial y)$$

When applying this vorticity component to the atmosphere, it is referred to as the *relative vorticity* of the horizontal flow because it does not include any rotation due to the rotation of the Earth, i.e., the coordinate system is assumed to rotate fixed to the Earth's surface. The magnitude of this relative vorticity is on the order of 10^{-4} per second. Cyclonic rotation gives positive values of relative vorticity while anticyclonic rotation is negative (in the Northern Hemisphere).

The rotation of the Earth contributes to the total or *absolute vorticity* of atmospheric flow. The Earth's vorticity is given by the Coriolis parameter:

$$f = 2*\Omega*\sin(\varphi)$$

where: Ω is the angular rotation of the Earth ($7.29*10^{-5}$/s) and φ is the latitude. The absolute vorticity (η) of the horizontal flow is the sum of the relative vorticity and the Coriolis parameter.

$$\eta = \zeta + f$$

If you are working with a gridded dataset, it is easy to calculate both relative and absolute vorticity for the grid. On the other hand, you can also qualitatively access the location of vorticity maxima and minima if you look at vorticity from a natural coordinate perspective.

In natural coordinates the relative vorticity is given by the expression:

$$\zeta = (V/r) - (\partial V/\partial n)$$

Where V is the magnitude of the wind speed along the streamline, and r is the radius of curvature. This expression shows that the vorticity value has a contribution from both the curvature of the flow (first term on the right-hand side of the equation) and the lateral wind shear (second term on the right-hand aide). These two terms allow you to qualitatively access the location of vorticity maxima by carefully examining the wind field.

First Term (rhs): The radius of curvature (r) is positive for cyclonic flow and negative for anticyclonic flow. The more curved is the flow, the smaller is the magnitude of r. Thus, for flat flow, r is large and the vorticity is relatively small. For highly curved flow, r is small and the vorticity is relatively large. This implies that a closed circulation will have a vorticity maximum or minimum in the center of the circulation.

Second Term (rhs): The term ($-\partial V/\partial n$) is the wind shear measured perpendicular to the flow. Starting at the axis of maximum wind speed, if you move in the positive n-direction, V will decrease and the term ($-\partial V/\partial n$) is positive. Thus, there is a positive contribution from this term on the left side of a jet stream axis (looking downstream) and a negative contribution on the right side.

The vorticity charts available to operational forecasters provide fields of absolute vorticity. In the atmosphere absolute vorticity is always positive, that is, f is greater than ζ. In looking at charts of absolute vorticity, positive relative vorticity centers will show up as maxima in the absolute vorticity field while negative relative vorticity centers will show up as minima.

Qualitative assessment can be used to anticipate the vorticity pattern associated with upper level troughs and jet streaks. A trough is an area of relatively lower height or pressure and cyclonically curved flow. As a result you would expect a lobe, or axis of maximum absolute vorticity, along the trough line. Similarly, an axis of minimum absolute vorticity can be found along a ridge line. Lobes along trough lines are usually better defined than lobes along ridge lines.

Let’s look at Figure 5-1. There is a trough over Iowa with a vorticity maximum and a lobe extending across the contour lines into Kansas. There is also a vorticity maximum with the low height centers off the California and the one in Ontario. In many situations where the trough is weak, i.e., it has small amplitude, the vorticity max or vorticity lobe, in combination with satellite data, is the best way to identify the feature location.

The jet stream is a broad band of winds with speeds in excess of 50 knots. It is identified using isotachs and has an axis of maximum wind that extends along the flow. Along this axis is an area of maximum wind called a jet streak. Consider the lateral wind shear along the axis. You would expect higher vorticity values on the left side of the axis (looking downstream) and relatively lower values on the right side. Figure 5-1 shows a typical jet axis west of the Pacific coast of Canada. There is a lobe of maximum vorticity along the flow from around 45^{o} north latitude east and east-northeast toward the British Columbia coast.

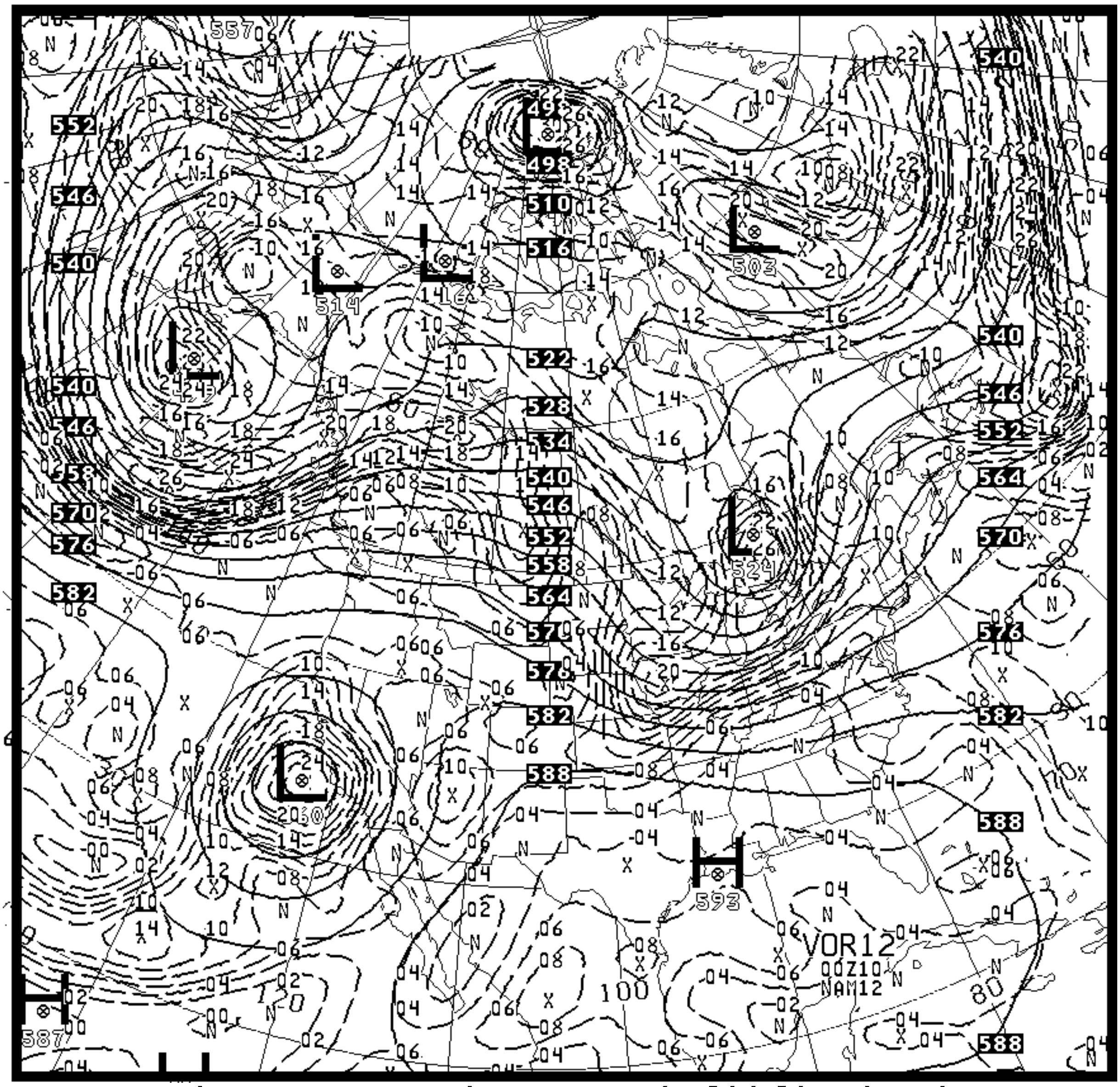

Figure 5-1: 500 mb contours (solid lines) and vorticity isopleths (dashed lines)

An area or lobe of higher vorticity values can help you identify troughs or jet streaks, depending upon whether the lobe is across the flow (with a trough) or along the flow (with a jet streak).

Divergence

Divergence (DIV) is another kinematic property of fluid flow and, in three dimensions, is expressed mathematically as:

$$DIV = \nabla \cdot V = \partial u/\partial x + \partial v/\partial y + \partial w/\partial z$$

where ∇ is the del operator and V is the velocity vector. DIV measures the expansion or spreading out of a velocity field.

In operational meteorology, due to the predominance of horizontal motion, divergence usually refers to the horizontal expansion (positive divergence) or contraction (negative divergence or convergence) of the wind field. Consider a cubic meter of air that is undergoing horizontal convergence. In this situation the horizontal flow is adding mass to fixed volume. One of two things can happen: (1) the density in the volume can increase; or (2) the inflowing air can flow out of the volume in the vertical direction. The latter tends to dominate in most situations. As a result, horizontal divergence is used to estimate or anticipate the vertical movement of air. This is expressed mathematically as the continuity equation. This equation is an expression for the conservation of mass.

If divergence is calculated at the Earth’s surface, it can be used to estimate the vertical motion at the top of the boundary layer. For example, divergence calculated from surface observations can provide a good estimate of where low level convective forcing might exist.

If divergence is calculated in the upper troposphere, with the assumption that minimal vertical motion occurs through the tropopause, it can be used to estimate the vertical motion from the mid-troposphere into the upper troposphere. McNulty (1978) correlated the mean divergence in the 300-200 mb layer with the occurrence of severe weather.

With the increased sophistication of computer forecast models, vertical motion is carried as an explicit variable. This reduces the need to estimate the vertical motion by other means. Nevertheless, for convective forecasting, identifying areas of surface convergence can be very useful for localizing convective initiation.

Advection

Advection is the transport of an atmospheric property by the wind. This transport process plays a very important part in local changes of temperature and moisture, among other atmospheric properties. Mathematically, advection is expressed as:

$$\text{Advection} = -(V\cdot\nabla)a$$

Where V is the vector velocity and the variable, a, is the property being transported.

Theoretically advection can be viewed as three dimensional. In operational meteorology the term advection is typically limited to transport by the horizontal flow while vertical transport is called convection. In Cartesian coordinates, horizontal advection is given by:

$$\text{Advection} = -[u*(\partial a/\partial x) + v*(\partial a/\partial y)]$$

Advection is better visualized by expressing advection in natural coordinates:

$$\text{Advection} = -V*(\partial a/\partial s)$$

Both of these expressions state that advection is the product of the wind speed in a given direction times the gradient of the property along that direction.

The vector expression for advection can be interpreted in terms of the relative orientation of the velocity vector and the property gradient vector. The property gradient vector mathematically points from lower to higher values. This is shown in Figure 5-2.

In meteorology, the minus sign is applied to the gradient vector, reversing the direction of that vector and pointing the gradient vector from higher toward lower values.

In Case A, the velocity vector and gradient vector are pointing in opposite direction. This gives a negative value to $(V\cdot\nabla)a$. Applying the minus sign gives a positive value. Thus, when the air flow is from higher to lower property values, the advection is positive.

In Case C, the velocity vector and gradient vector are pointing in same direction. This gives a positive value to $(V\cdot\nabla)a$. Applying the minus sign gives a negative value. Thus, when the air flow is from lower to higher property values, the advection is negative.

In Case B, the velocity vector and the gradient vector are perpendicular. In this case the dot product and advection value are zero. This is referred to as neutral advection.

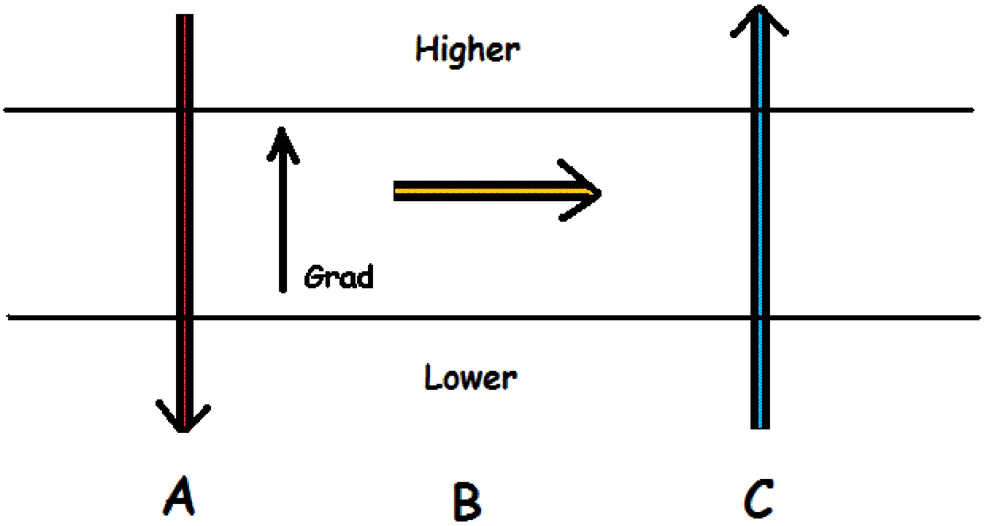

Figure 5-2: Interpretation of the Advection Equation; velocity vectors are shown in bold and color; the mathematical gradient vector is labeled "grad" while two isolines are used to show relatively higher and lower property values

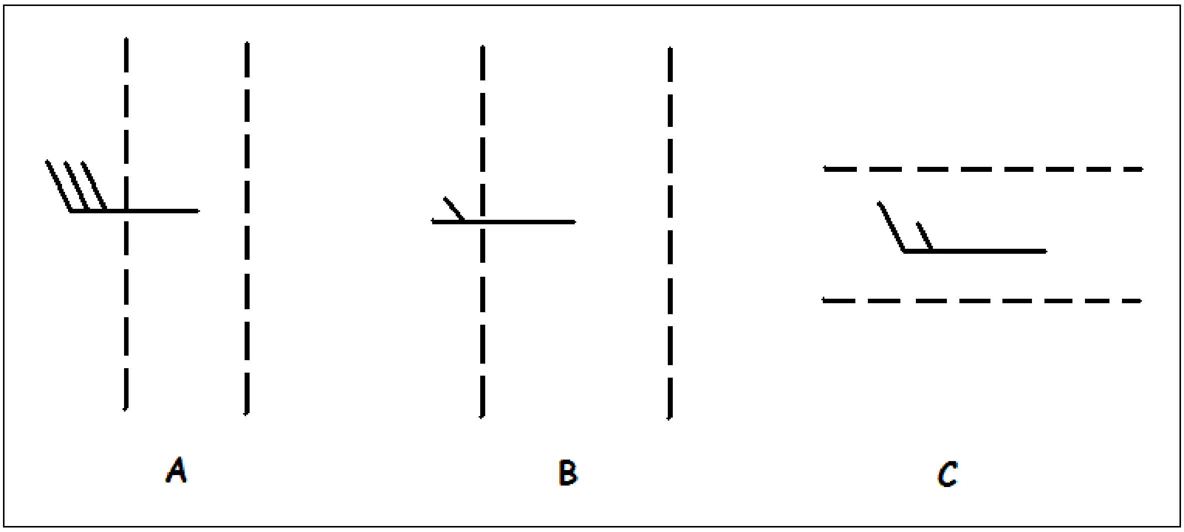

Figure 5-3: Schematic of Advection Intensities

Three general properties of advection are derived from these expressions. Advection is stronger when:

a. The wind speed is stronger.
b. The property gradient is stronger.
c. The flow is across the property isopleths.

Practical interpretation of advection is shown schematically in Figure 5-3. In Case A, the property gradient is relatively strong, the wind speed is relatively strong, and the flow is perpendicular to the isolines. This is an example of *strong advection*. In Case B, the property gradient is relatively weak, the wind speed is light, and the flow is perpendicular to the isolines. This is an example of *weak advection*. In Case C, the flow is along the isolines. This is zero or *neutral advection*.

If you need explicit values for advection, the Cartesian express can be evaluated from gridded data fields. If, however, you only need a qualitative assessment for advection, the practical application shown in Figure 5-3 can be easily adapted to standard level charts.

Advection can be evaluated for a variety of atmospheric parameters. For example, boundary layer temperature advection is a significant factor determining local temperature change (excluding the diurnal cycle due to radiation effects). Temperature advection is also a factor in determining vertical motion based on the omega equation (see Chapter 10). Positive temperature advection is referred to as warm air advection (WAA); negative temperature advection is called cold air advection (CAA). Vorticity advection is a factor in the omega equation. Low level moisture advection is something that is evaluated in thunderstorm forecasting and in assessing available moisture for winter storms.

Moisture Flux Divergence

Moisture flux divergence (MFD) has been used for decades by convective forecaster as a precursor of significant convection (Hudson, 1971). Operationally MFD is known as moisture convergence and is derived from the horizontal moisture and wind field. Remember that convergence is negative divergence. MFD is the gradient of the moisture flux (Vr) and is expressed mathematically as:

$$\nabla \cdot (Vr) = (V \cdot \nabla) r + (\nabla \cdot V) r$$

Where ▼ is the del operator, V is the velocity vector, and r is mixing ratio. MFD is composed of two terms: mixing ratio advection by the horizontal wind (first term, rhs); and the divergence of the horizontal wind times the mixing ratio (second term, rhs).

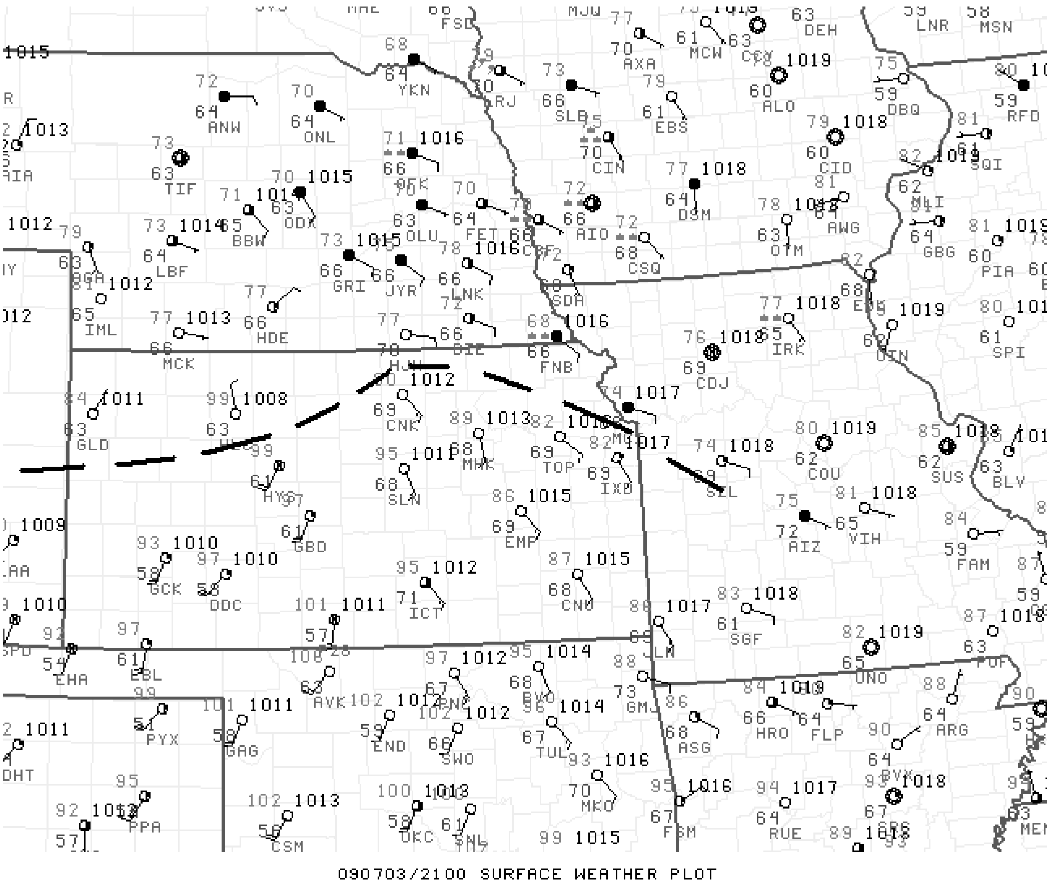

Figure 5-4a: Surface data for 2100Z, 03 July 2009
(*Source*: Storm Prediction Center)

A convective forecaster uses surface data to identify areas of mesoscale lift that are needed to initiate and sustain convective updrafts. Sufficient mass convergence at the Earth's provides a mechanism for this lift. Moisture flux divergence can be used to help identify these areas of lift. Experience has shown that in most situations, the convergence term dominates the advection term. This implies that minima in MFD (convergence) typically occur along some type of boundary. If, however, there is moderate to strong moisture advection (flow across a moisture gradient) in an area of weak convergence, you may see a minimum in MFD without a corresponding boundary. Bottom line: carefully examine the actual surface data when using MFD to see why there is a MFD maximum or minimum.

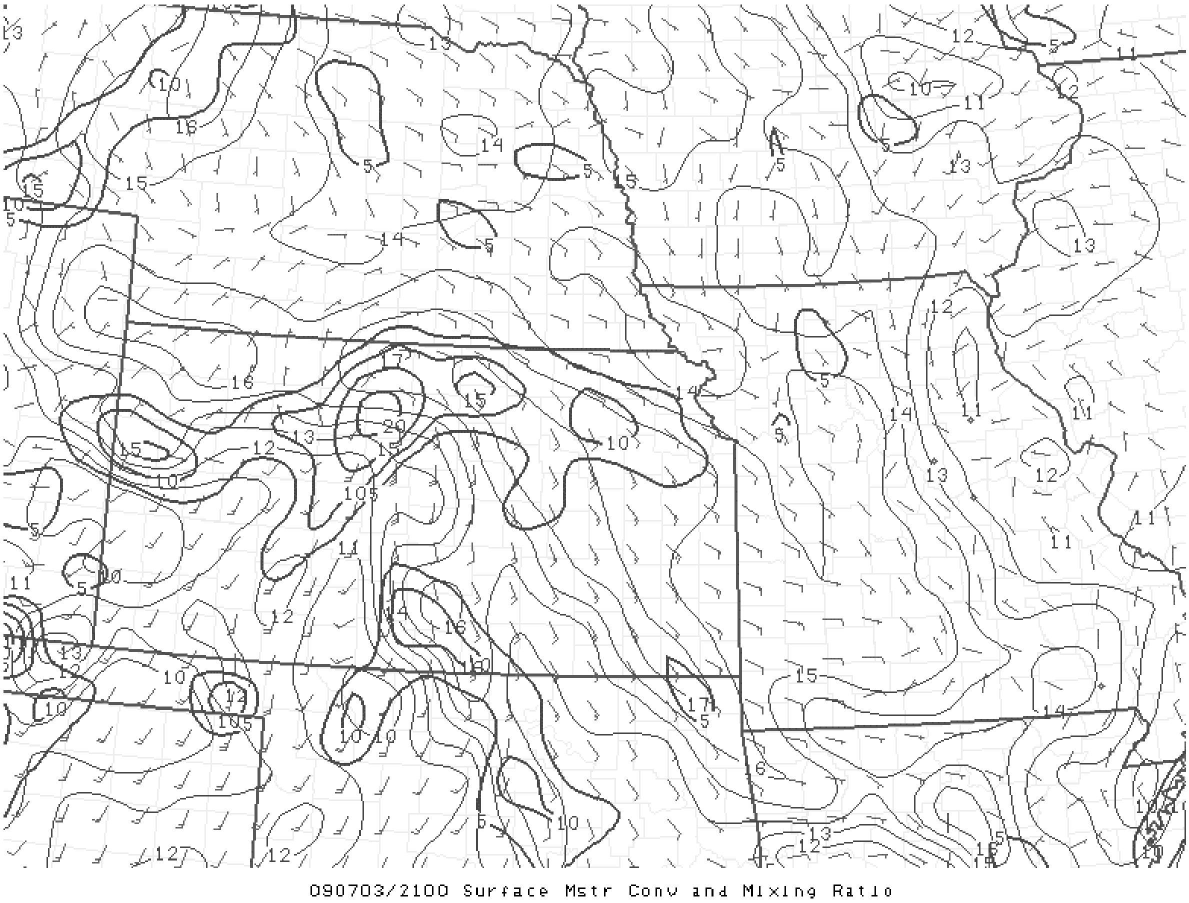

Figure 5-4b: Moisture Flux Divergence (heavy lines)and mixing ratio (light lines) for 2100Z, 02 July 2009 (*Source*: Storm Prediction Center)

Let’s look at an example. Figure 5-4a shows the surface data for Kansan and adjacent states. A boundary, shown as a bold dashed line, extends west-to-east from west central Kansas, into north central Kansas, and then eastward to the Kansas City area. South of this boundary there are southerly winds, temperatures in the upper 90s (Fahrenheit), and dew point temperatures mainly in the upper 60s, with a few upper 50s over southwest Kansas. To the north of the boundary winds are from the east to northeast, temperatures are in the 70s with dew point temperatures mainly in the 60s. The wind flow indicates good convergence along the boundary. Moisture advection appears relatively weak.

Figure 5-4b is the moisture flux divergence corresponding to Figure 5-4a. In this figure, convergence (negative divergence) is shown with positive values as bold, solid lines. There is a well-defined band of MFD along the boundary discussed above. The area of stronger MFD values in south-central Kansas and north-central Oklahoma is associated with a line of weak convergence in the wind flow.

Figure 5-4b also allows you to do a qualitative evaluation of the moisture advection by combining the wind information with the mixing ratio isopleths (light lines).

Geostrophic and Gradient Wind

The importance of geostrophic and gradient wind to operational analysis lies in the fact that these concepts relate the pressure field to the wind field. Thus, if you know the wind field, you can say something about the pressure field; if you know the pressure field, you can say something about the wind field. Because the practical implications of both geostrophic and gradient wind are the same, we discuss them together.

For geostrophic wind, there is a balance between the pressure gradient force and the Coriolis force. Geostrophic flow is straight, unaccelerated, frictionless flow. For gradient wind, an additional component (centripetal acceleration) is added to the balance to account for the curvature of the flow. For application in the boundary layer, friction must be also be added. The main effect of introducing friction is to turn the flow towards lower pressure in order to achieve balance among the various forces being considered.

The equation for geostrophic flow (in natural coordinates) illustrates the main analysis applications:

$$V_g = -(g/f)*(\partial p/\partial n)$$

Where:

- V_g = geostrophic wind speed
- g = acceleration of gravity
- f = Coriolis parameter (a function of latitude)
- p = pressure

In both geostrophic and gradient flow:

a. The tighter the pressure (or height) gradient, the stronger is the wind speed.
b. In the Northern Hemisphere, there is lower pressure (or height) to the left of the flow (looking downstream) and higher pressure (or height) on the right (Buy Ballot’s Law).
c. As you move poleward, the pressure (or height) gradient must increase to support the same geostrophic wind speed due to changes in the value of the Coriolis force.

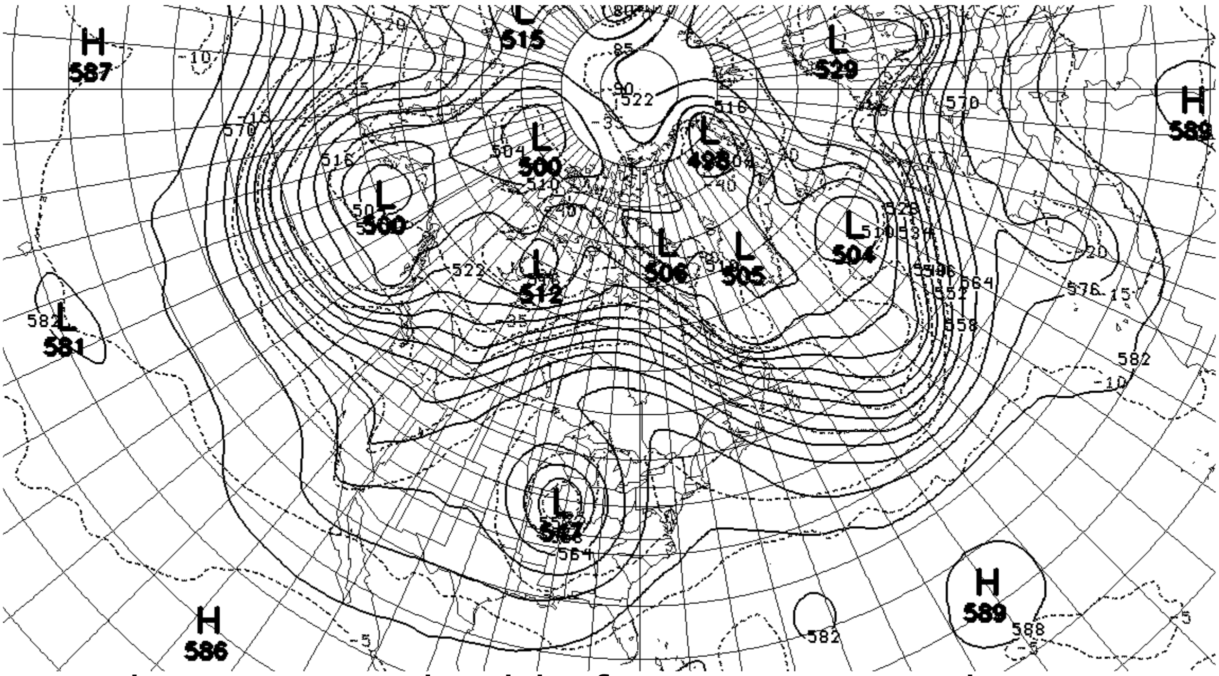

Figure 5-5: 500 mb Height for 00 UTC, 19 November 2009

From an analysis perspective you should be able to look at a surface pressure pattern or an upper level height chart and say something about both the direction of the flow as well as the relative speed along a latitude circle. For example, in Figure 5-5 the height pattern indicates cyclonic (counter-clockwise) flow around a low height center over the Illinois-Missouri border, southwest winds across the Canadian Prairie, and northwest flow from northern Quebec into the Canadian Maritimes. The tighter gradient over Indiana implies stronger winds in this area than over Colorado. The tighter gradients in the main westerly flow indicate where the main mid-latitude jet stream is likely to be found.

As a weather analyst you need to become comfortable looking at pressure and height patterns not only in terms of high and low centers, and troughs and ridges, but also in terms of cyclonic and anticyclonic flow as implied by the geostrophic and gradient concept.

Thickness

In operational meteorology thickness (h) refers to the vertical depth or distance between two pressure surfaces. The pressure surfaces bounding the layer of interest are typically standard pressure surfaces [i.e., 1000, 925, 850, 700, 500, 400, 300, 200, 250 or 200 mb]. In the old days thickness charts could be constructed by graphically subtracting the contour patterns from two standard level charts. With the transition to gridded data fields, a simple computer algorithm can produce a grid of thickness values for any layer of the atmosphere.

The hypsometric equation provides a theoretical basis for using thickness as a measure of mean temperature. This equation can be derived by integrating the hydrostatic equation from pressure level 1 to pressure level 2 (where level 1 is lower in height than level 2) after eliminating density using the ideal gas law.

$$h = z_2 - z_2 = (RT_m/g)*\ln(p_1/p_2)$$

Where:

z_i = height of pressure (p) level i
R = gas constant
g = acceleration of gravity
T_m = mean temperature of the layer from p_1 to p_2

This equation says that the thickness of the layer is proportional to the mean temperature of the layer. This gives the analyst a way of evaluating the mean thermal characteristics of various layers without directly examining temperature profiles. Typical thickness layers used in operational analysis include the following:

a. *1000-500 mb thickness*: rain-snow discrimination (5400 m isoline) and thermal gradient on the cold side of fronts.
b. *1000-850 mb thickness*: rain-snow discrimination (1300 m isoline) and rain/sleet/snow determination.
c. *850-700 mb thickness*: rain-snow discrimination (1540 m isoline) and rain/sleet/snow determination.

d. *1000-700 mb thickness*: rain-snow discrimination (2840 m isoline) and thermal gradient on the cold side of fronts.
e. *850-300 mb thickness*: propagation of mesoscale convective systems (MCS)

Use of thickness in precipitation type forecasting is discussed in more detail in Appendix B.

Concluding Remarks

This chapter has described six derived parameters that are very useful in analyzing the current and future state of the atmosphere. Many more derived parameters exist and are easily calculated from gridded datasets. These parameters provide a way of bringing theoretical concepts into operations. Use them wisely.

Draft: 11-28-2009
Final: 5-11-20111