

Chapter 4

Introduction to Objective Analysis

Atmospheric data are routinely collected around the world but observation sites are located rather randomly from a spatial perspective. On the other hand, most computer forecast models use some type of uniform grid for their calculations. If you are going to routinely use observed data for any type of finite difference calculations, there is a need to transform or interpolate these irregularly spaced observations to the uniform grid without human intervention. This process is called *objective analysis* (OA). This process is related to the subjective analysis process (line drawing and human interpretation) described in Chapters 1 and 3, but is totally free of human intervention.

In the late 1940s, as numerical weather prediction was getting started, the initial values for the model grids were determined by first subjectively drawing charts and then interpolating between isolines to determine grid values. From an operational perspective, this was a slow, labor-intensive process. OA was the answer. It eliminated the human from the process, was considerably faster, and likely more accurate.

The objective analysis processes described in this chapter focus on spatial interpolation. Temporal interpolation can also be used if you want to blend data over time onto a grid. Temporal interpolation is not addressed here.

Types of Objective Analysis

Objective analysis has evolved over the last 50 years and can be divided into three main categories.

Function Fitting Model: In this approach a set of observations is mathematically fit to a set of relatively smooth equations. Usually some type of least square error technique is used. An example of this approach is the polynomial expansion of Panosky (1949).

Successive Corrections Method: This approach is an iterative process. It starts with a first guess or background field. It then uses a linear weighting scheme to adjust the initial guess

and subsequent estimates to fit the grid point values to the observations to within a specified tolerance.

Statistical Interpolation Method (also called optimal or optimum interpolation): In this approach the ensemble average of the squared difference between the gridded and observed data is minimized using a least-square methodology.

The evolution of OA since the late 1950s has led to the current method for initializing forecast models: data assimilation. Data assimilation uses statistical interpolation methods that combine observations with short range forecasts to produce, as accurately as possible, a dataset that represents the current state of the atmosphere. The assimilation model produces a short forecast, usually 3 or 6 hours into the future, and then incorporates recent data into the model to adjust the model to the current observations.

The successive corrections approach will be discussed in detail in this chapter.

Empirical Linear Interpolation

Introduction: In this approach grid point values are determined as a linear combination of scalar variable values from nearby observing sites. The basic form of the linear equation is:

$$X_{\text{grid}} = \sum_{j=1 \text{ to } m} (h_j X_{oj})$$

where:

- X_{grid} = interpolated grid point value
- h_j = scalar weight at point j
- X_{oj} = observation at point j
- m = number of observations

The scalar weight has the general form:

$$h_j = w_j / [\sum_{j=1 \text{ to } m} (w_j)]$$

where:

- w_j = weighting function applied to point j

Note that the sum of all h_j equals one [$\sum(h_j)=1$].

The weight function (w_j) can be determined in any number of ways. Statistical functions are currently in vogue. This chapter will focus on historically and empirically derived functions. In general, empirically derived weight functions use distance

weighting where observations that are closer to the grid point are given more weight than observations that are farther away. Scale-selection properties can also be incorporated into these functions.

Practical Perspective: Ideally, all available observations can be used to determine the value at a grid point. If a weighting function falls off with distance, there is some distance away from the grid point where the weight becomes so small that the observations have a very little influence on the grid point value. To save computer time and eliminate these low-influence observations, only observations within a specified distance from the grid point are used in the calculation. This distance is called the "radius of influence." The size of the radius of influence also determines the degree of smoothing that is incorporated into the interpolation. As a general rule, smaller radii have less smoothing; larger radii create more smoothing.

Weighting Function Properties: Weighting functions should have the following properties:

- The influence of an observation should be higher for closer observations and lower for observations farther from the grip point.
- If an observation is coincident with a grid point, it should have a unit weight.
- The resulting grid values should be differentiable (a desirable property).
- The more observations included in the interpolation, the higher is the smoothing level.

Applying these properties result in a wide variety of weighting functions. The exact form of the weighting function is often determined by trial and error, hence the name, empirical.

An Example: Let's consider the example shown in Figure 4-1. There are 12 observations in the dataset. Only four observations are within the radius of influence: D, E, F and G. These will be used to calculate the value at the grid point (gp). Thus, for grid point, X_{gp} :

$$X_{gp} = (h_D * X_D) + (h_E * X_E) + (h_F * X_F) + (h_G * X_G)$$

Note that in this particular example, no consideration is given to the distribution of the observation about the grip point. Weighting functions can be designed with this in mind.

Let's define the weighting function, $w_j = \exp(-r) = e^{-r}$, for r less than a 3.5 units, where r is the distance from the grid point to the observation (in arbitrary units).

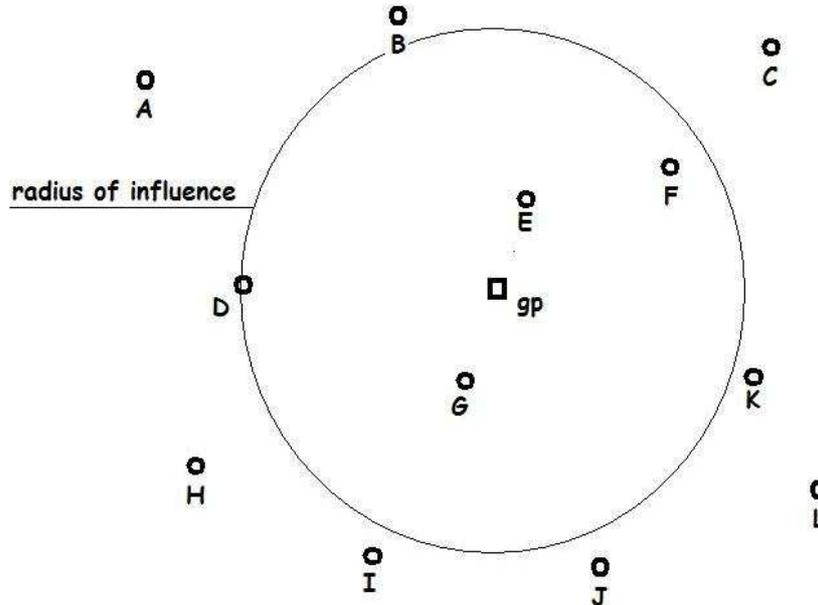


Figure 4-1: Example of Radius of Influence

The following table summarizes the calculations:

Point	$X_{oj}=Ob$	r	$w_j=e^{-r}$	h_j	h_j*X_j
D	74	3.35	0.0351	0.0536	3.97
E	73	1.25	0.2865	0.4375	31.94
F	69	2.80	0.0608	0.0928	6.40
G	77	1.30	0.2725	0.4161	32.04
Sum			0.6549	1.0000	74.35

The four observation values and corresponding distances from the observation point to the grid point are listed in the first three columns on the left. First, calculate the value of the weighting function. These values are shown in the w_j column. Then sum up the weighting functions for all observations. This value is at the bottom of the w_j column, 0.6549. Next, calculate the weight for each observation, h_j , by dividing individual weighting function value by the sum of the weight functions. Note that the sum of the weights adds up to one. Finally, multiply the observation values by the corresponding weight to get the

contribution of that observation to the final interpolated values. These values are shown in the last column on the right. The sum of the individual contributions gives interpolated values of 74.35.

You have now done one grid point. Are you ready to do the rest by hand or should we let a computer do it?

Successive Corrections

The empirical linear interpolation uses the actual observation values when calculating grid point values. The successive corrections approach begins with a first guess or background grid and then uses the difference between these background values and the observations to determine the final grid point values via an iterative process. This method is useful for grid areas where observations are sparse or non-existent.

The successive corrections procedure includes a series of steps:

- Estimate the background value at each observation point $[X_{bj}]$.
- Calculate the prediction error, that is, the difference between the background value at an observation point and the corresponding observation value $[X_{oj} - X_{bj}]$.
- Use empirical linear interpolation to distribute the prediction errors to the grid points.
- Correct the grid point value based on the distributed errors. $[X_{grid(k+1)} = X_{grid(k)} + \sum_{j=1 \text{ to } m} (h_j * \{X_{oj} - X_{bj}\})]$, where k is the iteration step index.
- Repeat the above steps until the value of all predicted errors is below a specified amount.

This iterative process allows the grid point values to converge to the observation values. At each iteration step the value of the weight can be modified or the radius of influence can be changed.

Cressman Scheme

The Cressman (1959) scheme is described here because it is the first operational successive corrections OA scheme used by the Weather Bureau (now the National Weather Service) in the early days of operational numerical weather prediction.

It is similar to the successive corrections scheme described in the previous section. It uses a non-zero weighting function

within a prescribed radius of influence. This radius of influence decreases with each successive scan. After a large number of scans the grid values converge to the observations. In areas of high observation density, the grid point values reflect the observations, while in low-density areas the grid values are closer to the first guess or background field.

The Cressman weighting function takes the form:

$$w_j = (d^2 - r^2) / (d^2 + r^2) \quad r < d$$

$$w_j = 0 \quad r \geq d$$

where:

- r = distance from the grid point to the observation
- d = radius of influence

This original weighting function was modified slightly and is referred to as the Cressman Model Extension:

$$w_j = [(d^2 - r^2) / (d^2 + r^2)]^\alpha \quad r < d$$

$$w_j = 0 \quad r \geq d$$

where $\alpha > 1$.

Selecting the proper values for d and α is somewhat empirical and depends upon the data spacing and the desired level of smoothing. In general, a smaller radius of influence has less smoothing; a larger radius of influence produces a higher degree of smoothing. For the original Cressman scheme a value for d that is approximately twice the average spacing of the observations tends to be a reasonable compromise between under-smoothing and over-smoothing.

See Thiébaux and Pedder (1987), page 94-99, for examples.

Barnes Scheme

The Barnes (1964) proposed a scheme which could be used to interpolate randomly spaced data onto a grid with a desired level of detail. The Barnes weighting function is given by:

$$w_j = \exp [-r^2 / 4k]$$

where:

- r = distance from the grid point to the observation
- k = parameter used to define the response of the weighting function

The value of k is chosen so that when $r=d$ (d is the radius of influence), the weighting function is e^{-4} times maximum value of data at $r=0$. The radius of influence is chosen so that it is greater than the average spacing of the observations.

The iterative process recommended by Barnes for determining grid point values of some variable is as follows:

- On the first scan, estimate the value of the variable at each grid point using the following scheme:

$$x_{i,j} = [(\sum_{n=1 \text{ to } N} w(r,d) * X_{on}) / \sum w(r,d)]$$

where:

- o $x_{i,j}$ = grid point value of the variable
- o X_{on} = observed value of the variable
- o N = number of observations
- o $w(r,d)$ = Barnes weighting function
- Next, estimate the variable value for each observation location by averaging the four closest grid values.
- Calculate the difference between the variable estimate and the observation value (residual).
- Distribute the differences using the linear interpolation scheme described above with the Barnes weighting function.
- Repeat the last three steps until the residual is less than a prescribed precision factor.

Smoothing

Smoothing is a process that averages data in space and/or time so that fluctuations of a scale smaller than a specified amount are reduced or removed. In some ways, smoothing is a type of OA.

Smoothing can be easily applied to a spatial field or time series. For example, a running mean is often used with time series to smooth out short-term fluctuations so that longer term cycles can be identified. For example, Figure 4-2 shows a time series of high temperature data for Kansas City International Airport (KMCI). The daily high temperatures are shown in blue and the 7-day running mean is in red. Even though day-to-day

fluctuations are large, the general warmer and cooler periods can be seen in the blue time series. The 7-day running mean damps out the small-scale changes and makes identification of the warmer and cooler period much easier. In fact, one could infer a periodicity of approximately two weeks.

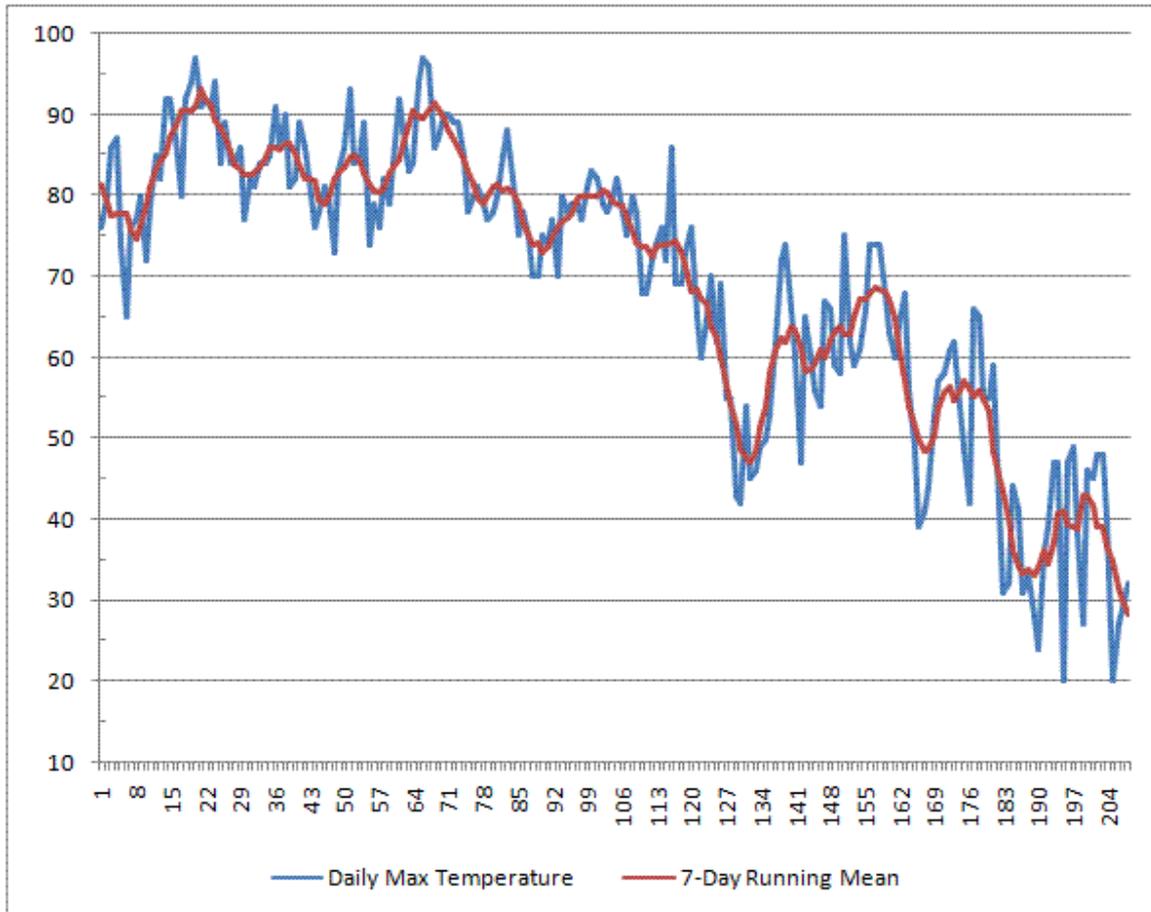


Figure 4-2: KMCI High Temperature Data

Smoothing schemes can take on a wide variety of forms. Details are beyond the scope of this chapter.

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